Project 1

Savings Plan

Introduction

In this project you will write a program that will compute the future balance of a savings account in which regular deposits are made. The calculations will incorporate inflation and adjust the results to 2013 dollars. The future value will be computed twice: once where the deposits are constant, so they decrease in value as a result of inflation, and again where the deposits are adjusted to take inflation into account.

Inflation

For example, suppose the person deposits $100.00 a month and that the inflation rate is 2% per month. In the first case, the value of the deposit at the end of the first month would be

$$\frac{100.00}{1.02} = 98.03921$$

and the value of the second deposit would be

$$\frac{100.00}{1.02^2} = 96.11688,$$

and so on. After 100 months, the value would be

$$\frac{100.00}{1.02^{100}} = 13.80330.$$

On the other hand, if the deposits are adjusted for inflation, then each deposit is larger than the previous deposit, so that they all have the same value in constant dollars. For example, the deposit at the end of the first month would be

$$100(1.02) = 102.00$$

and the deposit at the end of the second month would be

$$100(1.02)^2 = 104.04,$$

and so on. The deposit after 100 months would be

$$100(1.02)^{100} = 724.46.$$

Assumptions

We will make the following assumptions.

- The user deposits a fixed amount per month (interpreted first as inflated dollars and then as constant dollars).
- The inflation rate and the interest rate on the investment are constant over time.
- The user will make deposits for a whole number of years.
- The deposits are made at the end of each month.

Input and Output

Prompt the user for the following input:

- The monthly amount deposited.
- The number of years over which the amount is deposited.
- The interest rate as an annual percent.
- The inflation rate as an annual percent.

The output should be in two parts. First, if the deposits are not adjusted for inflation, then the deposits represent inflated dollars. Output

- The account balance after the specified number of years.
- The amount invested.
- The amount earned.

All output is in constant 2013 dollars.

The second part of the output is like the first part, except that the deposits are adjusted for inflation. In other words, the monthly deposits are steadily increased in order to keep up with inflation. Thus, every deposit represents the same value in 2013 dollars.

Calculations

Assuming that the deposits are not adjusted for inflation, the formula for the account balance after n payments is

$$A = \frac{P[(1+r)^n - 1]}{r(1+i)^n},$$

where A is the account balance, P is the monthly deposit, r is the monthly interest rate (as a decimal), i is the monthly inflation rate (as a decimal), and n is the number of monthly deposits.

The value of all the deposits is given by

$$\frac{P\left(1 - \frac{1}{(1+i)^n}\right)}{i}.$$

Note that the value of the first formula is undefined if $r = 0$ and the value of the second formula is undefined if $i = 0$. These are special cases that your program will have to test for. If $r = 0$, then the correct formula for A is

$$A = \frac{nP}{(1+i)^n}.$$

If $i = 0$, then the correct formula for the amount invested is nP.

On the other hand, assuming that the deposits are adjusted for inflation, the account balance after n deposits is

$$A = \frac{P(1+i)\left[\left(\frac{1+r}{1+i}\right)^n - 1\right]}{r - i}.$$

and the amount invested is nP.

Again, we see a possible problem. If $r = i$, then the value of the formula for A is undefined. In this case, the correct formula is $A = nP$.

In all cases, the interest earned is the future account balance minus the amount invested.

Be sure to follow the five steps discussed in class to develop your program. Work one example of each case by hand, using simple numbers. For example, you might work examples where

- $P = 100$, $n = 12$ (1 year), $r = 0.02$, and $i = 0.01$.
- $P = 100$, $n = 12$, $r = 0.02$, and $i = 0$.
- $P = 100$, $n = 12$, $r = 0$, and $i = 0.02$.
- $P = 100$, $n = 12$, $r = i = 0.02$.

Invoking Functions

You can see from the formulas that you will need the power function `pow()`, which was discussed in class. The website `http://www.cplusplus.com/reference/clibrary/cmath/` contains a list of all math functions in C++. You can click on any one of them to get a complete description, including examples. You may have to include the header file `<cmath>` in your program.

Decisions

You will have to make three independent decisions in your program: whether $r = 0$, whether $i = 0$, and whether $r = i$. You will have to use a separate `if` statement for each decision. Be sure to include in the body of the `if` statements only those formulas that depend on that condition.

Testing Your Work

Be sure to test your program throughly. Use the examples above that you calculated by hand first. Then try other cases. Test other special cases such as $r = i = 0$, $r < 0$, $i < 0$, $P = 0$, and $n = 0$.

Turning in Your Work

Place your file `SavingsPlan.cpp` in a folder named `Project 1` and drag the folder to the dropbox. Be absolutely sure that the file name and folder name are correct. Otherwise, I might not find your work. Your work is due by midnight, Wednesday, September 25, 2013. Do not underestimate the time it will take to complete this project. Get an early start!